Investigation of the Effects on Stability of Foot Rolling Motion
Based on a Simple Walking Model

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Abstract—The motion of the lower limbs in bipedal walking is characterized by a foot-rolling motion, including heel-strike and toe-off. In this paper, the dynamical influence of this motion on walking stability is examined using a simple walking model driven by a rhythmic signal from an internal oscillator. In order to model the rolling motion, a circular arc is attached to the tip of the legs. In particular, we obtained approximate periodic solutions and analyzed the dependence of the local stability on the circular arc radius using a Poincaré map, which revealed that the circular arc radius is optimal when it is similar in size to the leg length, to maximize the stable region for such characteristic parameters as mass ratio and walking speed. On the other hand, it is also found that a circular arc radius of zero maximizes the rate of convergence to the stable walking motion. These conflicting results imply that the optimal radius of a circular arc with respect to local stability exists from a trade-off between these different criteria, which should be considered in designing a biped robot.

I. INTRODUCTION

The motion of the lower limbs in human bipedal walking is characterized by a foot-rolling motion, including heel-strike and toe-off, where the feet appear to behave approximately like rigid circular arcs [1]. Many studies elucidated that the foot roll-over shape plays an important role generating effective walking, which has helped the design of such rehabilitation devices as prostheses [1], [14], [15].

Humans generate bipedal walking by manipulating their complicated and redundant musculoskeletal systems. This complexity makes it difficult to determine the essence of the walking dynamics. However, simple models of bipedal walking give great insight and facilitate the elucidation of the principles of the walking dynamics [2], [3], [16]. Therefore, simple mathematical models and simple biped robots have been used to investigate it. In particular, various simple biped robots with circular feet have been developed to examine the dynamical influences of the rolling foot. McGeer [17] originally used circular feet for passive dynamic walking and investigated their effects by numerical simulations. Following his research, many passive dynamic walkers employing circular feet were created and examined [11], [12]. Simple actuated biped robots demonstrated that the circular foot improves such properties as energy efficiency and walking speed [9], [16], [19]. Wisse et al. [20] pointed out the stability improvement due to the circular foot. However, they didn’t perform a thorough analysis to find clear criteria and guiding principles regarding stability in designing the circular foot.

In this paper, we investigate the dynamical effects on stability of a foot rolling motion in bipedal walking. We examine it based on a simple walking model with circular feet driven by a rhythmic signal from an internal oscillator, where the oscillator periodically moves the legs. Our previous work [7] clarified that when the walking model has no circular feet, that is, it has pointed feet, it has a self-stabilization property depending on such characteristic parameters as mass ratio and walking speed. In this paper, we obtained approximate periodic solutions of the walking motion and eigenvalues of the Jacobian matrix of the Poincaré map to clearly elucidate the dynamical influences on the local stability. Analytical and numerical results reveal that the circular arc radius is optimal when it is similar in size to the leg length to maximize the stable region for the characteristic parameters. On the other hand, it is also found that the radius of zero maximizes the rate of convergence to the stable walking motion. These conflicting results imply that an optimal radius with respect to stability exists from the trade-off between these different criteria, which should be considered in designing a biped robot.

II. SIMPLE WALKING MODEL

Figure 1 shows a schematic model of a simple walking model composed of a body and two legs whose length is \( l \). The legs are connected at the hip joint. Body mass \( M \) and leg mass \( m \) are concentrated at the hip joint and at \( l \) distant from the hip joint, respectively. A circular arc whose radius is \( k \) \((k \geq 0)\) is attached to the tip of each leg. The circular arc of the stance leg rotates on the ground without slip. This model is constrained on the \( x-y \) plane and walks to the \( x \)-direction. Acceleration due to gravity is \( g \).

This model has two degrees of freedom, \( \theta_1 \) and \( \theta_2 \), where \( \theta_1 \) is the angle of the stance leg relative to the perpendicular line to the ground subject to the geometric constraint of the circular arc not to slip relative to the ground, and \( \theta_2 \) indicates the angle of the swing leg relative to the stance leg directly controlled by torque \( u \).

The step cycle of the walking motion consists of two types of successive phases, single-supported and double-supported phases, where only the stance leg and both legs, respectively, are in contact with the ground.
Fig. 1. Schematic model of simple walking model

A. Single-supported phase

In the single-supported phase, dimensionless equations of motion for \( \dot{\theta} = [\dot{\theta}_1 \dot{\theta}_2] \) are given by

\[
D(\theta)\ddot{\theta} + C(\dot{\theta}, \dot{\theta}) + G(\theta) = U
\]

where \( D(\theta) = \{D_{ij}\} \) \((i,j = 1,2)\) is the inertia matrix, \( C(\dot{\theta}, \dot{\theta}) = [C_1 C_2]^T \) is the nonlinear term, \( G(\theta) = [G_1 G_2]^T \) is the gravity term, and \( U = [U_1 U_2]^T \) is the input torque term. Specifically,

\[
D_{11} = (1-k)^2 + k^2 + 2(1-k)kc\theta_1 + \beta((1-k-a)^2 + (1-k)^2 + 2k^2 + a^2 + 2(2-2k-a)k\theta_1 - 2kac(\theta_2 - \theta_1) - 2(1-k)ac\theta_2)
\]

\[
D_{12} = D_{21} = -\beta a\{a - kc(\theta_2 - \theta_1) - (1-k)ac\theta_2\}
\]

\[
D_{22} = \beta a^2
\]

\[
C_1 = -\{1 - k + \beta(2 - 2k - a)\}k\dot{\theta}_1^2 s\theta_1
\]

\[
-\beta a(k\dot{\theta}_2 - \dot{\theta}_1)^2 s(\theta_2 - \theta_1) - \beta(1-k) a\dot{\theta}_2 (\theta_2 - 2\dot{\theta}_1) s\theta_2
\]

\[
C_2 = -\beta(1-k) a\dot{\theta}_2^2 s\theta_2
\]

\[
G_1 = -(1-k)s\theta_1 - \beta((2-2k-a)s\theta_1 + as(\theta_2 - \theta_1))
\]

\[
G_2 = \beta a s(\theta_2 - \theta_1)
\]

\[
U_1 = 0, \quad U_2 = u'
\]

where \( c = \cos x, s = \sin x, \beta = m/M, u' = u/Mgl, \quad \tau = t\sqrt{g/l}, \) and \( t \) indicates time. From now on, \( () \) implies the derivative with respect to \( \tau \).

This model has an internal oscillator that generates the kinematics as investigated in our previous works [4–8]. Specifically, angle \( \theta_2 \) is controlled using a high-gain feedback control to follow the desired angle generated by a rhythmic signal from the oscillator. The oscillator, whose phase is \( \phi \), creates constant rhythm \( (\phi = \omega = const.) \). Since the motion of the swing leg is modeled as a pendulum-like oscillation [18], desired angle \( \theta_{2d} \) is designed as a simple sinusoidal function of the oscillator’s phase \( \phi \), given by

\[
\theta_{2d} = S \cos \phi
\]

where \( S \) indicates the parameter that determines the stride. Actuator torque \( u' \) is given by

\[
u' = -K_p(\theta_2 - \theta_{2d}) - K_d(\dot{\theta}_2 - \dot{\theta}_{2d})
\]

where \( K_p \) and \( K_d \) are appropriate gain constants. In light of the above description, state variable \( q \) of this model is defined as 

\[
q = [\theta_1 \dot{\theta}_1 \theta_2 \dot{\theta}_2]
\]

B. Double-supported phase

When the circular arc of the swing leg lands on the ground, both of the legs are in contact with the ground. This geometric condition is given by \( r(q) = 2\theta_1 - \theta_2 = 0 \). We assume that the time duration of the double-supported phase is sufficiently short, so that immediately following the double-supported phase the swing leg is constrained on the ground and the stance leg leaves the ground. This means that the role of each leg changes in an infinitesimal period, which is written by

\[
\begin{bmatrix}
\dot{\theta}_1^+ \\
\dot{\theta}_2^+
\end{bmatrix} = Q \begin{bmatrix}
\theta_1^+ \\
\theta_2^+
\end{bmatrix}, \quad Q = \begin{bmatrix}
1 & -1 \\
0 & 1
\end{bmatrix}
\]

Impulsive force occurs at the contact point of the circular arc of the swing leg, resulting in discontinuous changes in angular velocities. We also assume that the stance leg leaves the ground without interaction and that the influence of the actuator during the double-supported phase is too small and ignored (see Sec. III-A for details). Under these assumptions, by calculating the impulse added to the walking model (see e.g. [13]) and considering the geometric constraint of the circular arc of the support leg just before and after the double-supported phase, the relationship between the states immediately prior to and following the double-supported phase is given by

\[
\begin{bmatrix}
\dot{\theta}_1^- \\
\dot{\theta}_2^-
\end{bmatrix} = Q \begin{bmatrix}
\theta_1^- \\
\theta_2^-
\end{bmatrix}
\]

where \( (^-) \) and \((^+) \) indicate just before and after the double-supported phase, respectively. This yields the following relation from the condition of desired angles \( \theta_{2d}^- = -\theta_{2d}^+ \)

\[
\phi^+ = \phi^- - \pi
\]
III. APPROXIMATE ANALYSIS

A. High-gain feedback control

Angle $\theta_2$ is manipulated by a feedback control in (3). When we use the sufficiently high-gain feedback control torque, angle $\theta_2$ can catch up with desired angle $\theta_{2d}$ in a sufficiently small period of time. Here, we introduce parameters $\omega_l$ and $\zeta_l$ for feedback gains $K_p$ and $K_d$ in (3) by

$$K_p = D_{22} \omega_l^2, \quad K_d = 2D_{22} \zeta_l \omega_l$$

During the single-supported phase, rise time $\delta$ with respect to controlled angle $\theta_2$ becomes $\mathcal{O}(\omega_l^{-1})$ from equations of motion (1). Therefore, when rise time $\delta$ is much shorter than the step period ($= \mathcal{O}(1)$) by using a large value of $\omega_l$ and an appropriate value of $\zeta_l$, the error between angle $\theta_2$ and desired angle $\theta_{2d}$ can be ignored. Thus, we assume that angle $\theta_2$ is identical to desired angle $\theta_{2d}$ during the single-supported phase by considering $\delta \ll 1$.

As described above, we also assumed that time interval $\varepsilon$ for the double-supported phase is sufficiently small, which resulted in discontinuous changes in angular velocities. This leads to a discrepancy between the angular velocities of angle $\theta_2$ and desired angle $\theta_{2d}$. Therefore, we assume that time interval $\varepsilon$ is much shorter than rise time $\delta$, that is, $\varepsilon \ll \delta$. By following [13], the equations of motion for state variable $\ddot{q}^t = [\dot{\theta}_1 \dot{\theta}_2 x y]$ during the double-supported phase are written by

$$D_e(q_e) \ddot{q}_e + C_e(q_e, \dot{q}_e) + G_e(q_e) = U_e + F_e$$

where $x$ and $y$ represent the positions of the center of the circular arc of the support leg relative to the ground, $D_e(q_e) \ddot{q}_e$ is the inertia term, $C_e(q_e, \dot{q}_e)$ is the nonlinear term, $G_e(q_e)$ is the gravity term, $U_e$ is the input torque term, and $F_e$ indicates the dynamical influence due to impulsive force at the contact point. Note that these equations don’t incorporate the geometric constraint of the circular arc. Since the impulsive force results in changes only in velocities, we can express $q_e = \mathcal{O}(1)$, $\dot{q}_e = \mathcal{O}(1)$, $\ddot{q}_e = \mathcal{O}(\varepsilon^{-1})$. In light of the above description, we can express $U_e = \mathcal{O}(\delta^{-1})$.

By integrating equations of motion (8) from the start to the end of the double-supported phase, terms $D_e(q_e) \ddot{q}_e$ and $F_e$ become $\mathcal{O}(1)$, terms $C_e(q_e, \dot{q}_e)$ and $G_e(q_e)$ become $\mathcal{O}(\varepsilon)$, and term $U$ becomes $\mathcal{O}(\varepsilon/\delta) \ll \mathcal{O}(1)$. Therefore, terms $C_e(q_e, \dot{q}_e)$, $G_e(q_e)$, and $U_e$ are ignored. That is, the input torque can be ignored during the double-supported phase, and relationship (6) is obtained.

Therefore, in this paper we analyze the walking motion under the condition $\delta \ll 1$. In this case, the state variable is redefined as $q^t = [\dot{\theta}_1 \dot{\theta}_2 \phi]$. In this case, the state variable is redefined as $q^t = [\dot{\theta}_1 \dot{\theta}_2 \phi]$. The numerical simulation was carried out for feedback gains $K_p$ and $K_d$ in (3) by

$$\dot{q} = f(q), \quad q^c = h(q^c)$$

$$f(q) = \begin{bmatrix} \lambda \dot{\theta}_1 - \beta \lambda \dot{\theta}_2 (\dot{\phi}) - \dot{\theta}_{2d}(\phi) \{1 + 2\beta(1-a)^2\} \\ \omega \end{bmatrix}$$

$$h(q^c) = \begin{bmatrix} \dot{\phi} + \beta \lambda \dot{\theta}_2 (\phi) \\ \dot{\phi} - \pi \end{bmatrix}$$

and $\lambda = \frac{1 - k + 2\beta(1-k-a)}{1 + 2\beta(1-a)^2}$. Note that $\lambda < 0$ means that this system has a mechanical oscillatory behavior without torque input and foot contact.

According to parameter $\lambda$, periodic solutions are obtained as follows:

**Case 1. $\lambda > 0$**

Periodic solutions to (9) are given by

$$\theta_1(\tau) = \theta_1(0) + \int_0^\tau \cos^\lambda \phi(\tau') \cos^\lambda \phi(\tau') d\tau'$$

$$\phi(\tau) = \omega \tau, \quad 0 \leq \tau \leq T$$

where $\xi = \beta \lambda (1 - a)^2 / (1 + 2\beta(1-a)^2)$ and $\lambda = 0$ and $\tau = T(= \pi / \omega)$ imply the dimensionless time just after and before a double-supported phase, respectively. Figure 3(a) shows the periodic solutions obtained by this approximate analysis and the numerical simulation by using $\beta = 0.2$, $k = 0.5$, $a = 0.5$, $S = 0.1$, $\omega = 1.0$, $\omega = 10\pi$, and $\zeta = 0.8$. Note that the numerical simulation was carried out based on original nonlinear equations (1), (4), (5), and (6). Figure 3(c) shows the stick diagram, where bold and dashed lines indicate the stance and swing legs, respectively.
Periodic walking motion is asymptotically stable if all the eigenvalues of Jacobian matrix $J(q^*)$ are inside the unit circle on the complex plane, that is, all the magnitudes of the eigenvalues are less than 1.

By following [10], Jacobian matrix $J(q^*)$ becomes equivalent to the product of three matrices $B$, $D$, and $E$ shown below. First, let the periodic solution, step period, and perturbed state from periodic solution $q^*(\tau)$ from just after the $i$th double-supported phase to the next double-supported phase be $q^*(\tau)$, $\tau^*$, and $q^*(\tau) + \hat{q}_i(\tau)$, respectively, where $\hat{q}_i(\tau)$ is subject to the initial condition $\hat{q}_i(0) = \hat{q}^+_i$. Then, matrices $B$ and $D$ are given by

$$B = D_q h(q^*(\tau^*))$$
$$D = I - \frac{\hat{q}_i^+(\tau^*) D_q r(q^*(\tau^*))^T}{D_q r(q^*(\tau^*))^T \hat{q}_i^+(\tau^*)} \tag{16}$$

where $I$ is a $3 \times 3$ unit matrix and $D_q = \frac{\partial}{\partial q}$. The evolved perturbation after the $i$th double-supported phase $\hat{q}^+_{i+1}$ satisfies

$$\hat{q}_{i+1}^+ = B D \hat{q}_i(\tau^*) \tag{17}$$

The substitution of perturbed state $q^*(\tau) + \hat{q}_i(\tau)$ into the equations of motion (9) gives

$$\dot{\hat{q}}_i(\tau) = D_q f(q^*(\tau)) \hat{q}_i(\tau) \tag{18}$$

Matrix $E$ is derived by integrating (18) as follows:

$$\dot{\hat{q}}_i(\tau) = E \hat{q}_i^+ \tag{19}$$

We investigate the local stability of the walking motion by using the approximate periodic solutions obtained in the previous section. The substitution of the periodic solutions to (16) and (19) gives matrix $BDE$ by

$$BDE = \begin{bmatrix} 0 & * & 0 \\ * & 1 + (\xi - \eta) \omega^2 d & -\xi \omega \{2 + (\xi - \eta) \omega^2 d\} + \eta \omega \\ * & -\omega d & 1 + \xi \omega^2 d \end{bmatrix} \tag{20}$$

where $d = \frac{1 - \cosh \sqrt{\lambda T}}{\lambda \sqrt{\lambda T}}$ for $\lambda > 0$ and $d = \frac{1 + \cosh \sqrt{\lambda T}}{\lambda \sqrt{\lambda T}}$ for $\lambda < 0$ and $\eta = \beta a (1 - a) / (1 + \beta (1 - a)^2)$. Matrix $BDE$ has one zero eigenvalue ($\Lambda_1 = 0$), and the other two eigenvalues $\Lambda_{2,3}$ are obtained from equation

$$\Lambda^2 + (\zeta - 2) \Lambda + 1 = 0 \tag{21}$$

where $\zeta = (\eta - 2 \xi) \omega^2 d$. If $\lambda > 0$, $\zeta > 0$ is always satisfied. Equation (6) follows $\Lambda_{2,3} = \frac{1}{2} (2 - \zeta \pm \sqrt{\zeta^2 - 4})$. Therefore, the stability depends on these eigenvalues as follows:

Case 1. $\Lambda_{2,3}$ are complex conjugate

In this case, it follows $0 < \zeta < 4$ and the periodic solutions are marginally stable from

$$\Lambda_2 \Lambda_3 = \bar{\Lambda} \Lambda = |\Lambda|^2 = 1 \tag{22}$$

Case 2. $\Lambda_{2,3}$ are real and distinct

In this case, it follows $\zeta > 4$ or $\zeta < 0$ and the periodic solutions are unstable from

$$\Lambda^2 \leq 1 \tag{23}$$

**Case 2. $\lambda < 0$**

Periodic solutions to (9) become equivalent to

$$\theta_1(\tau) = S \left( \frac{1}{2} + \xi \right) \frac{\cos \sqrt{-\lambda T} - \cos \sqrt{-\lambda} (\tau - T)}{1 - \cos \sqrt{-\lambda T}} - S \xi \cos \omega T$$
$$\phi(\tau) = \omega T \quad 0 \leq \tau \leq T \tag{11}$$

Figure 3(b) displays the periodic solutions by using $\beta = 0.2$, $k = 1.5$, $a = 0.5$, $S = 0.1$, $\omega = 1.0$, $\omega_T = 10\pi$, and $\zeta = 0.8$.

**Case 3. $\lambda = 0$**

Periodic solutions to (9) are obtained by

$$\theta_1(\tau) = S \left( \frac{1}{2} + \xi \right) \left( 1 - \frac{2 \tau}{T} \right) - S \xi \cos \omega T$$
$$\phi(\tau) = \omega T \quad 0 \leq \tau \leq T \tag{12}$$

Note that it is confirmed that in both cases $\lambda > 0$ and $\lambda < 0$, periodic solution $\theta_1(\tau)$ uniformly converges to (12) as $\lambda$ approaches 0. That is, $\lim_{\lambda \rightarrow +0} \theta_1(\tau) = \lim_{\lambda \rightarrow -0} \theta_1(\tau)$.

**C. Local stability analysis regarding characteristic parameters**

In this section, we investigate the local stability of the walking motion by using a Poincaré map. We use the state just after a double-supported phase as the state on the Poincaré section. The Poincaré map, which is the return map from one point on the Poincaré section to the next point on the Poincaré section, is denoted as $q \mapsto p(q)$, then

$$\hat{q}^+_{i+1} = p(q^+_i) \tag{13}$$

where $q^+_i$ is the state immediately following the $i$th double-supported phase. Note that fixed point $q^*$ on the Poincaré section satisfies

$$q^* = p(q^*) \tag{14}$$

By adding perturbation $\hat{q}^+_i$ from fixed point $q^*$ just after the $i$th double-supported phase, where $(\cdot)$ indicates the perturbation, and linearizing Poincaré map $p$ at fixed point $q^*$, Jacobian matrix $J(q^*)$ of the Poincaré map satisfies

$$\hat{q}^+_{i+1} = J(q^*) \hat{q}^+_i \tag{15}$$
Case 3. $\Lambda_{2,3}$ are degenerate

In this case, it follows $\zeta = 0, 4$ and the periodic solutions are marginally stable from

$$\Lambda_{2,3} = 1$$

(24)

Thus, the periodic solutions are marginally stable for $0 \leq \zeta \leq 4$ and otherwise they are unstable. It should be noted that although this analysis only reveals that the walking motion is marginally stable or unstable, we can find that the walking motion is actually asymptotically stable when this analysis indicates marginal stability as investigated in [7], which we thoroughly confirmed through numerical simulations.

Since parameter $\zeta$ depends on mass ratio $\beta$, circular arc’s radius $k$, and angular velocity (walking speed) $\omega$, these parameters determine the walking stability. Figure 4 shows the stability region with respect to $k$ and $\omega$ for various $\beta$, divided into two stable and two unstable regions by two boundaries that we also verified by numerical simulations based on original nonlinear equations. As shown by a dot in this figure, for radius parameter $k$, the vicinity of $k = 1$ has the largest stable region with respect to walking speed $\omega$, which slightly depends on $\beta$ and $\omega$. This suggests that when $k$ is close to 1, the structure of this model closely resembles a circular disc, resulting in a specific increase of the stable region.

In Fig. 4, when $k$ exceeds the value at which the stability boundary intersects with axis $\omega = 0$ and that is less than 1, indicated by an open dot, $\lambda$ becomes less than 0. When $\lambda > 0$, the stability region for walking speed $\omega$ is simply divided into one stable and one unstable region and they monotonically change depending on $k$. On the other hand, when $\lambda < 0$, the stability region becomes complex. When $\lambda > 0$, this dynamical system is divergent, that is, it falls forward. The foot contact and change of the leg roles enable it to establish stable periodic walking. On the contrary, when $\lambda < 0$, it becomes oscillatory. This model has an internal oscillator that generates basic rhythm and makes the swing leg moves periodically. Therefore, when $\lambda < 0$, the relationship between these two rhythms has dynamical influences on the walking stability, resulting in the complex stability region.

Let such $k$ that has the largest stable region regarding $\beta$ and $\omega$ as shown in Fig. 4 be $\bar{k}$. Figures 5(a) and (b) show the stability region with respect to $\beta$ and $\omega$ for $k < \bar{k}$ and $k > \bar{k}$, respectively, divided into two stable and one unstable regions by two boundaries. When $k < \bar{k}$, the increase of $k$ results in the increase of the stable regions. On the other hand, when $k > \bar{k}$, the increase of $k$ leads to the decrease of the stable regions.

IV. STABILITY REGARDING THE RATE OF CONVERGENCE

The previous section clarified the dynamical contribution of the foot circular arc to the walking stability for such characteristic parameters as mass ratio and walking speed by the stability region. However, in addition to the stability regions for parameters, it is important to elucidate how quickly the walking motion converges to the stable periodic motion after it is disturbed from the periodic motion. Although we can find the rate of convergence from the eigenvalue analysis based on the Jacobian matrix of the Poincaré map, the previous section based on approximate analysis didn’t clarify it. Therefore, to show the dynamical effects of the foot rolling motion for more details, we examine it in this section.

When the walking motion is small, the approximate analysis establishes sufficiently similar results to the rigorous numerical analysis, as shown in Figs. 3(a) and (b) and Fig. 4. Therefore, we investigate the rate of convergence by rigorous numerical simulations based on original nonlinear equations. Specifically, we find a fixed point on the Poincaré section by using the Newton-Raphson method and add numerical perturbations to calculate the Jacobian matrix and its eigenvalues.
where we find the fixed point to an accuracy of $10^{-7}$ and add perturbations of the order of $10^{-4}$.

Figure 6(a) shows the maximum eigenvalue between the eigenvalues of the Jacobian matrix with respect to $k$ and $\omega$ by using $\beta = 0.2$, where the stability region is almost the same as obtained by approximate analysis in the previous section.

Figure 6(b) displays the maximum eigenvalue versus $k$ for various $\omega$. They reveal that the increase of walking speed $\omega$ results in the decrease of the maximum eigenvalue and that the increase of radius parameter $k$ results in the increase of the maximum eigenvalue. This means that although the stability increases as the walking speed increases, the stability decreases as the foot circular arc’s radius increases.

Although the stability analysis in the previous section revealed that the increase of $k$ up to $k$ results in the increase of the stable region for characteristic parameters, this analysis clarified that the increase of $k$ causes the decrease of the rate of convergence. These results imply that the foot rolling motion has conflicting effects on the walking stability, suggesting that there is an optimal circular arc radius with respect to stability from a trade-off between these different criteria.

V. CONCLUSION

In this paper, we investigated dynamical effects on walking stability of the foot rolling motion based on a simple walking model with circular arc at the tip of each leg driven by a rhythmic signal from an internal oscillator. In particular, we examined the stability region for such characteristic parameters as mass ratio and walking speed, and the rate of convergence to the stable walking motion. Regarding the stability region for parameters, stable region increases as the circular arc radius increases, and the radius is optimal when it is similar in size to the leg length. On the other hand, the rate of convergence decreases as the radius increases and the radius of zero is optimal. These results may conflict with each other, implying that an optimal radius of the circular arc with respect to stability exists from a trade-off between different criteria. Because this paper is only based on local stability analysis, it requires more thorough analysis such as basin of attraction and global stability analysis to fully clarify the dynamical effects.

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